

From Cooper-pair glass to unconventional superconductivity : a unified approach to cuprates and pnictides

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Abstract – We report a microscopic model wherein the unconventional superconductivity emerges from an incoherent ‘Cooper-pair glass’ state. Driven by the pair-pair interaction, a new type of quasi-Bose phase transition is at work. The interaction leads to the unconventional coupling of the quasiparticles to excited pair states, or ‘super-quasiparticles’, with a non-retarded energy-dependent gap. The model describes quantitatively the quasiparticle excitation spectra of both cuprates and pnictides, including the universal ‘peak-dip-hump’ signatures, and for the pseudogap phase above T_c . The results show that instantaneous pair-pair interactions account for the SC condensation without a collective mode.

Despite its wide applications, the BCS theory [1] fails to account for the physical properties of a large variety of high- T_c superconductors (SC), the cuprate family, but also the more recent iron-based superconductors. A striking feature of these materials is the proximity to an insulating phase, whether anti-ferromagnetic (cuprates), spin density wave (iron based SC, Bechgard salts) or localization (ultra-thin films). Just beyond the insulating phase, the SC dome appears in the phase diagram as a function of carrier concentration between two critical points. Understanding the transition from such an insulating to SC state is still a major challenge.

Microscopic measurements reveal an unconventional quasiparticle (QP) dispersion, the ‘peak-dip-hump’ structure [2], often attributed to the coupling to a collective mode [3–8]. Although the peak to dip energy follows both the neutron resonance and T_c as a function of doping [6,9,10], the finer shape of the QP spectra and their temperature dependence remain a challenge. Moreover, in the temperature range $[T_c, T^*]$ a pseudogap (PG) state persists, having a Fermi-level gap Δ_p much larger than the critical energy scale $k_B T_c$ in cuprates (see [11] and ref. therein) and also in iron-based SC [12–14].

In this letter, these questions are addressed within the

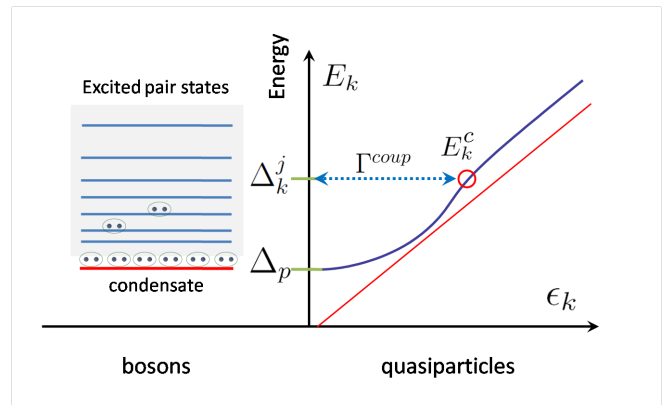


Fig. 1: Illustration of the boson-fermion PPI model [15]. The SC ground state (at Δ_p) has two distinct types of excitations: a distribution Δ_k^i of pair (boson) excitations, left panel, and quasiparticle (fermion) excitations. We demonstrate the strong coupling of the condensate QP of energy E_k^c to pre-existing excited pair states of equal energy Δ_k^j . The composite object is called a ‘super-quasiparticle’.

pair-pair interaction model (PPI). We show that the main unconventional features of high- T_c SC can be understood

in a microscopic theory wherein incoherent pairs in the *Cooper-glass state* interact to form the coherent superconducting state. As a result of this novel PPI, the quasiparticles become coupled to the excited pair states (see Fig. 1). These ‘super-quasiparticles’ give rise to an unconventional excitation spectrum wherein the gap function in the SC state is energy dependent but non-retarded. The theory is in full agreement with the experimental spectra on cuprates and pnictides, despite the order of magnitude variation in the energy gap.

The results point to a universal mechanism in high- T_c driven by the interaction between pairs, giving key physical quantities such as the condensation energy and elementary excitations, as a function of temperature and doping. In particular, the ‘peak-dip-hump’ originates from instantaneous electron interactions, thus discarding a bosonic mode as its origin in these materials.

Microscopic model. The hamiltonian describes normal electrons coexisting with interacting preformed pairs:

$$H = H_0 + H_{pair} + H_{int} \quad (1)$$

where the first term H_0 describes the normal metal phase, and the second term is the pairing hamiltonian:

$$H_{pair} = - \sum_i \sum_k (\Delta_k^i b_k^{i\dagger} + \Delta_k^{i*} b_k^i) \quad (2)$$

Here $b_k^{i\dagger}$ creates the i th pair state as composites of two fermions: $b_k^i = a_{-k\downarrow}^\dagger a_{k\uparrow}^\dagger$, and of binding energy Δ_k^i .

The first two terms $H_{PG} = H_0 + H_{pair}$ describe a non-superconducting state, a Cooper-pair glass having no global phase, formed by the superposition of pairs in random states. SC coherence is achieved due to the pair-pair interaction term giving rise to the characteristic DOS (Fig. 2, red curve):

$$H_{int} = \frac{1}{2} \sum_{i \neq j} \sum_{k, k'} \beta_{k, k'}^{i, j} b_{k'}^j b_k^{i\dagger} + h.c. \quad (3)$$

where $\beta_{k, k'}^{i, j}$ are the coupling coefficients, which we later tie to β^c , the SC order parameter.

Cooper-pair glass state. The accumulated results of photoemission [16], local tunneling experiments [17–19] and normal coherence length [20] imply a scenario in which, contrary to BCS theory, the pseudogap is linked to some form of precursor pairing [21]. The existence of Fermi-surface arcs just above T_c , as seen using ARPES [16], is further evidence. Without the PPI ($H_{int} = 0$) we consider that the system consists of incoherent preformed pairs with an energy distribution:

$$P_0(\Delta^i) \propto \frac{\sigma^2}{(\Delta^i - \Delta_0)^2 + \sigma_0^2} \quad (4)$$

where Δ_0 and σ_0 are the average gap and the half-width, respectively.

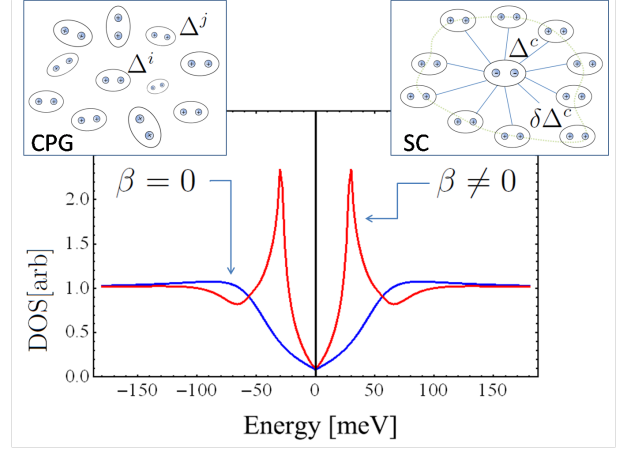


Fig. 2: DOS in the Cooper-pair glass (CPG) state, blue curve, showing a broad pseudogap of width $2 < \Delta_i > = 2\Delta_0$ and no coherence peaks. DOS in the SC state, red curve, with pronounced dips due to the QP excited-pair coupling.

In the spinor notation: $\tilde{a}_k^i = (a_{k\uparrow}^i, a_{-k\downarrow}^{i\dagger})$, the equation of motion is: $i\hbar \frac{d\tilde{a}_k^i}{dt} = [\tilde{a}_k^i, H_{PG}] = H_{PG}^i \tilde{a}_k^i$, where H_{PG}^i is the effective matrix:

$$H_{PG}^i = \begin{pmatrix} \epsilon_k & -\Delta_k^i \\ -\Delta_k^i & -\epsilon_k \end{pmatrix} \quad (5)$$

The latter is diagonal in the quasiparticle basis: $\tilde{\gamma}_k^i = \Lambda_k^i \tilde{a}_k^i$ with eigenvalues, $E_k^{i\pm} = \pm\sqrt{\epsilon_k^2 + \Delta_k^{i2}}$, leading to: $H_{PG} = \sum_i \sum_k \tilde{\gamma}_k^{i\dagger} (E_k^i \sigma_z) \tilde{\gamma}_k^i$, where σ_z is the standard Pauli matrix. In the continuum limit the spectral function $A_{PG}(k, E)$ acquires a significant width [22] and the $T = 0$ DOS becomes a convolution:

$$N_{PG}(E) = N_n(E_F) \int_0^\infty d\Delta^i P_0(\Delta^i) \frac{E}{\sqrt{E^2 - \Delta^{i2}}} \quad (6)$$

As a result of the pair distribution, the coherence peaks in the DOS are absent (blue curve, Fig. 2), a key feature of the incoherent *Cooper-pair glass*. This state is intimately related to the pseudogap observed once SC coherence is lost, i.e. at T_c or within a vortex core.

Equations of motion with H_{int} . Adding the term $[\tilde{a}_k^i, H_{int}]$ to the equation of motion, we obtain:

$$\begin{aligned} i\hbar \dot{a}_{k\uparrow}^i &= \epsilon_k a_{k\uparrow}^i - \Delta_k^i a_{-k\downarrow}^{i\dagger} + \sum_{j, k'}' \beta_{k, k'}^{i, j} b_{k'}^j a_{-k\downarrow}^{i\dagger} \\ i\hbar \dot{a}_{-k\downarrow}^{i\dagger} &= -\epsilon_k a_{-k\downarrow}^{i\dagger} - \Delta_k^i a_{k\uparrow}^i + \sum_{j, k'}' \beta_{k, k'}^{i, j} b_{k'}^{j\dagger} a_{k\uparrow}^i \end{aligned} \quad (7)$$

Obviously, without pairing ($\Delta^i = 0$), electrons are uncoupled from holes, reflecting the normal state. To the contrary, the second (anomalous) terms in (7) are generated by the removal of an electron-pair by a hole or a hole-pair by an electron (Fig. 3, middle panel). The third term is

new: the final state now contains a fermion triplet, which we call ‘super-anomalous’. For a fixed (j, k) , a *quadrone* of zero spin and charge is annihilated leaving a pair plus a fermion (Fig. 3, lower panel).

Since the i th electron (hole) is also coupled to all $j \neq i$, the hamiltonian cannot be simply diagonalized in terms of a set of quasiparticle operators $\{\tilde{\gamma}_k^i\}$. However, the fermion operator triplet can be decoupled by the quantum average of pair permutations:

$$b_{k'}^j, a_{-k\downarrow}^{i\dagger} \simeq \langle a_{-k'\downarrow}^j a_{k'\uparrow}^j \rangle a_{-k\downarrow}^{i\dagger} + \langle a_{-k\downarrow}^{i\dagger} a_{-k'\downarrow}^j \rangle a_{k'\uparrow}^j + \langle a_{k'\uparrow}^j a_{-k\downarrow}^{i\dagger} \rangle a_{-k'\downarrow}^j \quad (8)$$

resulting in the equation of motion:

$$i\hbar \frac{d\tilde{a}_k^i}{dt} = (H_{PG}^i + \delta\Delta_k^i \mathcal{J}) \tilde{a}_k^i + \sum_{j,k'}' \beta_{k,k'}^{i,j} \left[\Gamma_{k,k'}^{i,j}(\uparrow\uparrow) \tilde{a}_{k'}^j + \Gamma_{k,k'}^{i,j}(\uparrow\downarrow) \tilde{a}_{k'}^{j\dagger} \right] \quad (9)$$

in which the two Γ -matrix coefficients are:

$$\Gamma_{k,k'}^{i,j}(\uparrow\uparrow) = \begin{pmatrix} \langle a_{-k\downarrow}^{i\dagger} a_{-k'\downarrow}^j \rangle & 0 \\ 0 & \langle a_{k'\uparrow}^j a_{k'\uparrow}^{i\dagger} \rangle \end{pmatrix}$$

$$\Gamma_{k,k'}^{i,j}(\uparrow\downarrow) = \begin{pmatrix} 0 & \langle a_{k'\uparrow}^j a_{-k\downarrow}^{i\dagger} \rangle \\ \langle a_{-k'\downarrow}^j a_{k'\uparrow}^{i\dagger} \rangle & 0 \end{pmatrix}$$

and $\mathcal{J} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. These equations display coupling coefficients depending on different states (k, k') and different pairs (i, j) , where two are *spin aligned*, $\Gamma(\uparrow\uparrow)$, and two are *spin reversed*, $\Gamma(\uparrow\downarrow)$. The vertex amplitudes imply instantaneous electron/hole interactions, all inherent to the super-anomalous term of Fig. 3. The new correction to the gap function is:

$$\delta\Delta_k^i = \sum_{j,k'}' \beta_{k,k'}^{i,j} \langle b_{k'}^j \rangle \quad (10)$$

which follows directly from (7) with: $b_k^j \rightarrow \langle b_k^j \rangle$.

Quasiparticle coupling. In order to define Bogoliubov quasiparticles in such a system, we apply the basis transformation which diagonalizes the first two terms on the r.h.s. of equation (9):

$$\bar{\mathcal{O}} = \bar{\Lambda}_k^i \mathcal{O} \bar{\Lambda}_k^{i-1}$$

for a general operator \mathcal{O} . Writing the quasiparticle basis as $\tilde{\gamma}_k^i = \bar{\Lambda}_k^i \tilde{a}_k^i$, yields:

$$i\hbar \frac{d\tilde{\gamma}_k^i}{dt} = \bar{\Lambda}_k^i (H_{PG}^i + \delta\Delta_k^i \mathcal{J}) \bar{\Lambda}_k^{i-1} \tilde{\gamma}_k^i = \bar{E}_k^i \sigma_z \tilde{\gamma}_k^i \quad (11)$$

It is important to stress that the eigenvalues (\bar{E}_k^i) depend on the *modified gap function*; their dispersion is: $\bar{E}_k^i =$


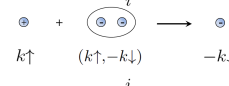
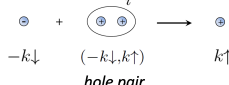
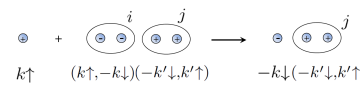
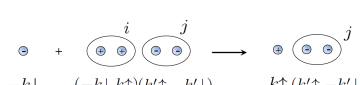
Fermion	 $k\uparrow$ hole	$\epsilon_k a_{k\uparrow}^i$ normal
Quasiparticle	 $k\uparrow + (k\uparrow, -k\downarrow) \rightarrow -k\downarrow$	$-\Delta_k^i a_{-k\downarrow}^{i\dagger}$
	 $-k\downarrow + (-k\downarrow, k\uparrow) \rightarrow k\uparrow$ hole pair	$-\Delta_k^i a_{k\uparrow}^i$ anomalous
Super-quasiparticle	 $k\uparrow + (k\uparrow, -k\downarrow)(-k'\downarrow, k'\uparrow) \rightarrow -k\downarrow(-k'\downarrow, k'\uparrow)$	$\beta_{k,k'}^{i,j} b_{k'}^j a_{-k\downarrow}^{i\dagger}$
	 $-k\downarrow + (-k\downarrow, k\uparrow)(k'\uparrow, -k'\downarrow) \rightarrow k\uparrow(k'\uparrow, -k'\downarrow)$ quadrone	$\beta_{k,k'}^{i,j} b_{k'}^{j\dagger} a_{k\uparrow}^i$ super-anomalous

Fig. 3: Summary of the various fermion/hole terms encountered in the ‘exact’ equation of motion (7). Upper panel: normal state electron/hole of energy ϵ_k . Middle panel: (anomalous term) standard BCS-type processes. Lower panel: (super-anomalous term) an electron (hole) annihilates a *quadrone*, having zero spin and zero charge, leaving a *super-quasiparticle* consisting of a pair plus a fermion. The associated coupling energy is $\beta^{i,j}$.

$\sqrt{\epsilon_k^2 + (\bar{\Delta}_k^i)^2}$, where $\bar{\Delta}_k^i = \Delta_k^i - \delta\Delta_k^i$, which we note is first order in β .

Using the $\bar{\Lambda}_k^i$ transformation, the equations of motion (9) can now be written in terms of QP operators:

$$i\hbar \frac{d\tilde{\gamma}_k^i}{dt} = \bar{E}_k^i \sigma_z \tilde{\gamma}_k^i + \sum_{j,k'}' \beta_{k,k'}^{i,j} \left[\bar{\Gamma}_{k,k'}^{i,j}(\uparrow\uparrow) \tilde{\gamma}_{k'}^j + \bar{\Gamma}_{k,k'}^{i,j}(\uparrow\downarrow) \tilde{\gamma}_{k'}^{j\dagger} \right] \quad (12)$$

As a result of the PPI, the second term of the full equation of motion (12) contains the coupling of the i th quasiparticle to all other quasiparticles $j \neq i$, with the QP-QP coupling proportional to the $\bar{\Gamma}$ coefficients. It implies that quasiparticles interact via pair states and conversely that pair states interact via quasiparticles: a novel QP-pair vertex is thus revealed.

To illustrate the effect of the coupling we focus on the case where, for wave vectors k and k' , only two quasiparticles of energy E_k^i and $E_{k'}^j$ become degenerate (higher degeneracies are possible) and the exact operators satisfy: $i\hbar \frac{d\tilde{\gamma}_k^i}{dt} = i\hbar \frac{d\tilde{\gamma}_{k'}^j}{dt} = E_{kk'}^{ex} \tilde{\gamma}_k^i$. In the bi-spinor basis $\tilde{\gamma}_k^j = (\tilde{\gamma}_k^j, \tilde{\gamma}_k^{j\dagger})$, a new object of dimension 4 in the a_μ fermions, the secular equation is obtained:

$$\begin{pmatrix} (E_{kk'}^{ex} \mathbf{1} - \bar{E}_k^i \sigma_z) \cdot \mathbf{1} & -\beta L_{k,k'}^{i,j} \\ -\beta L_{k',k}^{j,i} & (E_{kk'}^{ex} \mathbf{1} - \bar{E}_{k'}^j \sigma_z) \cdot \mathbf{1} \end{pmatrix} \times \begin{pmatrix} \tilde{\gamma}_k^i \\ \tilde{\gamma}_k^{i\dagger} \end{pmatrix} = 0 \quad (13)$$

with $L^{i,j}$ a matrix of dimension 4:

$$L_{k,k'}^{i,j} = \begin{pmatrix} \bar{\Gamma}_{k,k'}^{i,j}(\uparrow\uparrow) & \bar{\Gamma}_{k,k'}^{i,j}(\uparrow\downarrow) \\ \bar{\Gamma}_{k,k'}^{i,j}(\uparrow\downarrow) & \bar{\Gamma}_{k,k'}^{i,j}(\uparrow\uparrow) \end{pmatrix} \quad (14)$$

The analogy with the lowest order pairing matrix H_{PG}^i , Eq. (5), is striking. In the conventional BCS theory, electron (ϵ_k) and hole ($-\epsilon_k$) states are coupled via the pair potential Δ ; here the PPI ($\sim \beta$) leads to the coupling of the QP states ($\bar{E}_k^i, \pm \bar{E}_{k'}^j$). Since the determinant of the secular matrix must vanish, we obtain:

$$(E_{kk'}^{ex2} - \bar{E}_k^{i2})(E_{kk'}^{ex2} - \bar{E}_{k'}^{j2}) = \beta^4 \det(L^{j,i} \cdot L^{i,j}) \quad (15)$$

where the explicit QP-QP coupling $\sim \beta^4 \Gamma^4$, appearing on the r.h.s., is assumed to be small but finite.

The exact eigenstates $E_{kk'}^{ex}$ correspond to a new super-quasiparticle, $(\tilde{\gamma}_k^i, \tilde{\gamma}_{k'}^j)$, and thus to the quadron $(\bar{\Delta}_k^i, \bar{\Delta}_{k'}^j)$. While the \bar{E}_k^i are to first order in the PPI $\propto \beta$, the coupling involved in the super-quasiparticle is to higher order in $\beta \Gamma$. Since the latter is small, the coupling of the quasiparticles $\tilde{\gamma}_k^i$ and $\tilde{\gamma}_{k'}^j$ need only be considered at the *degeneracy point*:

$$\bar{E}_k^i = \bar{E}_{k'}^j \quad (16)$$

while otherwise, \bar{E}_k^i and $\bar{E}_{k'}^j$ are uncoupled. This degeneracy condition thus plays a central role in the theory.

Superconducting gap function. The SC ground state can be derived from the mean-field expression (10) wherein all pairs are assumed to be degenerate. The final-state gap function is thus written:

$$\bar{\Delta}_k = \Delta_{k,0} - \delta \Delta_k^{i=c} \quad (17)$$

where $i = c$ indicates pairs of the condensate and $\Delta_{k,0} = < \Delta_k^i >$. As in our previous work Refs. [15], we take the interaction to be proportional to the DOS of preformed pairs: $\beta_{k,k'}^{i,j} = g_k g_{k'} P_0(\Delta_k^i) P_0(\Delta_{k'}^j)$, where g_k takes into account the d -wave pairing. The crucial point is that all the pairs $\Delta_{k'}^j$ are degenerate in the condensate.

The gap equation (17) must be self-consistent for zero kinetic energy, wherein the QP states are at the Fermi level. Thus, setting $\epsilon_k = 0$ and $\bar{\Delta}_k = \Delta_k^c$, yields:

$$\Delta_k^c = \Delta_{k,0} - 2 \beta_k^c P_0(\Delta_k^c) \quad (18)$$

where $\beta_k^c = \frac{N_{oc}}{2} g_k \sum_{k'} g_{k'} P_0(\Delta_{k'}^c) < b_{k'}^c >$ is the mean-field *condensate* pair-pair interaction and N_{oc} , the number of pairs ($N_{oc} \gg 1$). Since the mean-field parameter β^c is proportional to $N_{oc}(T)$, as a result of the quasi-Bose transition, the second term represents the *condensation energy*. As the temperature rises, it gradually decreases and finally vanishes at T_c , contrary to the spectral gap [26] – a clear departure from conventional SC.

A key aspect of the problem is that the gap function Δ_k^c in equation (18) must be modified for non-vanishing kinetic energy, where a quasiparticle *becomes degenerate*

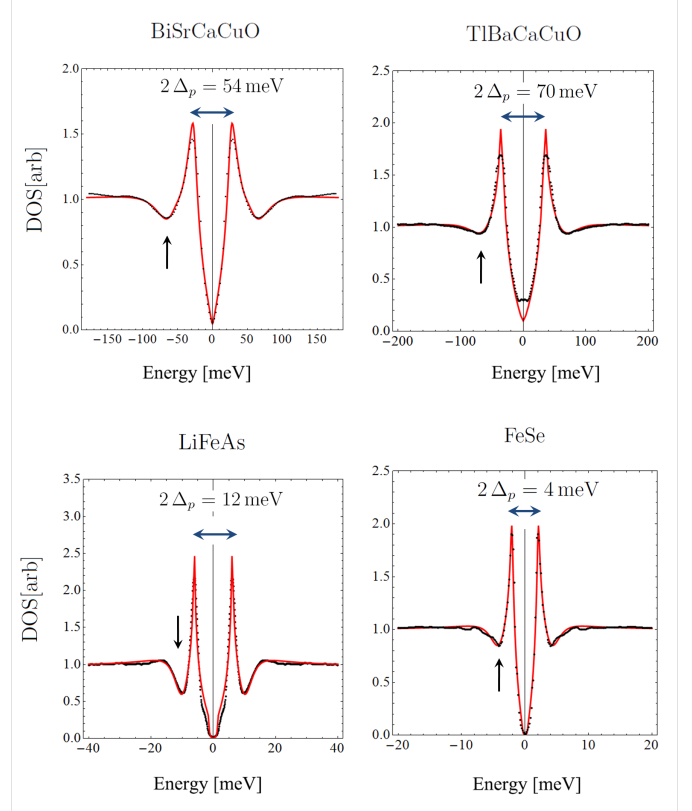


Fig. 4: Fits to the tunneling DOS of 4 very different SC materials using the same gap function (20). We compare the cuprate with iron-based materials: BiSrCaCuO (slightly overdoped with $\Delta_p = 27$ meV), TlBaCaCuO ($\Delta_p = 35$ meV), LiFeAs ($\Delta_p = 6$ meV), FeSe ($\Delta_p = 2$ meV) taken from Refs. ([7, 23–25]) respectively. The other numerical values used for the fits ($\beta^c, \Delta_0, \sigma_0$) are summarized in Table I. The dip position, indicated by the arrow in each case, follows approximately: $E_{dip} \simeq \Delta_p + 2\beta^c$.

with an excited pair state (see Fig. 1). The latter coupling energy $\sim \beta^2 \Gamma^2$ is to second order while the renormalized gap function, proportional to β^c , remains large. Thus, for excited states, $\epsilon_k > 0$, the gap equation (17) is:

$$\bar{\Delta}_k^{i=ex} = \Delta_{k,0} - 2 \beta_k^c P_0(\Delta_k^{i=ex}) \quad (19)$$

where both $\Delta_{k,0}$ and β_k are assumed independent of ϵ_k . Recalling equation (16), the *correct degeneracy point* is $\Delta_k^{i=ex} = \bar{E}_k$ where we identify $i \rightarrow \Delta_k^{i=ex}$ as the excited pair, degenerate with the state $j \rightarrow \bar{E}_k = \sqrt{\epsilon_k^2 + (\bar{\Delta}_k)^2}$ of the condensate. Dropping the overbar, the full gap equation for excited states, reads:

$$\Delta_k(E_k) = \Delta_{k,0} - 2 \beta_k^c P_0(\sqrt{\epsilon_k^2 + \Delta_k(E_k)^2}) \quad (20)$$

We thus have an energy dependent and self-consistent equation for the gap function which leads to a strictly non-hyperbolic QP dispersion and, most significantly, gives rise

SC parameters		BiSrCaCuO	TlBaCaCuO	LiFeAs	FeSe
spectral gap	Δ_p	27	35	6	2
pair-pair int.	β^c	19.5	11.5	2.4	.95
dist. maximum	Δ_0	48.5	52.5	7.6	3.2
dist. width	σ_0	24.5	32	4	2

Table 1: Numerical values (all in meV) obtained from the fits of Fig. 4 for the 4 different materials indicated. Note the order of magnitude difference between BiSrCaCuO and FeSe and yet the same basic parameters apply. In all cases, we find that $\Delta_0 \sim \Delta_p + \beta^c$ and $E_{dip} \simeq \Delta_p + 2\beta^c$, where E_{dip} is the dip position.

to the dip in the spectral function (Fig. 2). One can now identify its physical origin: the strong coupling of the SC quasiparticle with excited pair states.

Comparison with experiments. The instantaneous interactions in the hamiltonian imply that the DOS can be calculated with *no retardation effects* in the Green's function. If no quasiparticle lifetime effect is invoked, at $T = 0$, the DOS for the d -wave condensate can be calculated by the standard formulae using $\frac{\partial \epsilon_k}{\partial E_k}$:

$$\begin{aligned}
N_{SC}^d(E) &= \mathcal{N}_n(E_F) \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^\infty d\epsilon_k \delta(E_k - E) \\
&= \mathcal{N}_n(E_F) \int_0^{2\pi} \frac{d\theta}{2\pi} \left[\frac{E_k - \Delta_k(E_k, \theta) \frac{\partial \Delta_k}{\partial E_k}}{\sqrt{E_k^2 - \Delta_k(E_k)^2}} \right]_{E_k=E} \quad (21)
\end{aligned}$$

where $\mathcal{N}_n(E_F)$ is the normal DOS at the Fermi energy.

The SC DOS is thus proportional to the derivative of the gap function (20) wherein the peak-dip-hump is due to the interaction term: $\frac{\partial \Delta_k(E_k)}{\partial E_k} = 2\beta_k^c \frac{dP_0(E_k)}{dE_k}$. The controlling parameters are thus the pair-pair interaction β^c and the condensate pair number, N_{oc} , but the distribution $P_0(E_k)$ plays an essential role. Since the derivative has two extrema, the first one reinforces the QP coherence peaks, giving them an unconventional wide shape, while the second extremum produces the dip [27]. As in our previous work [15], the DOS (21) can be used to fit a wide variety of tunneling spectra of high T_c superconductors with remarkably few parameters: β^c , the mean pair-pair interaction, Δ_0 and σ_0 which characterize the distribution of pair states.

Among the cuprates, the tunneling characteristics of BiSrCaCuO (2212) or BiSrCaCuO (2203) have been the most clearly established (see [2] and references therein). Much success has recently been done on iron-based SC (see [10] and references therein), such as BaKFeAs, doped Fe(Se,Te) [6], as well as LiFeAs [7] or FeSe [25] where typical spectra are shown in Fig. 4. Since we focus on the SC aspects of the DOS, the background density is removed and the spectra symmetrized, without affecting adversely the SC gap and peak-dip-hump features. Along with a slightly overdoped BiSrCaCuO [23], Fig. 4 shows a recent high-quality spectrum on a 3-layer TlBaCaCuO [24], in-

dicating the universality of the peak-dip-hump features. In the same figure, the spectra are fitted using the PPI model.

The parameters of the fits are given in Table I. First, we note the relatively sharp peaks at $eV = \pm\Delta_p$ in the iron-based SC as compared to BiSrCaCuO and TlBaCaCuO. Indeed, higher peaks are quite rare, partly due to thermal smearing at 4.2 K but also due to a finite quasiparticle lifetime, which we estimate to be ~ 1.5 meV in the case of BiSrCaCuO and an order of magnitude less for FeSe. The detailed shape of all the spectra are accurately fitted using the same gap function (20) in the DOS, from energies within the gap, to the wide QP peaks and the pronounced dip. The parameters thus have the same meaning despite the range of values, and the very different composition and structure of the materials.

We find that $E_{dip} - \Delta_p \simeq 2\beta^c$, where $\beta^c \simeq 2k_B T_c$ follows the SC dome but without the collective mode scenario. Rather, it emerges from the novel QP-pair vertex inherent to the super-anomalous term of the equation of motion (7). These super-quasiparticles cause the dip in the spectrum and signal the long range SC order. The condensate PPI energy β^c depends on the product of the pairing amplitude Δ_p with the carrier density p : $\beta^c(p) \propto p \times \Delta_p$, providing a simple explanation for the SC dome. The mechanism is thus the interplay between the pair binding energy, decreasing with p , and the number of pairs increasing with p .

Conclusion. We propose a scenario for high- T_c superconductors wherein the initial incoherent state is the Cooper-pair glass, whose properties explain the observed pseudogap and Fermi-arc phenomena in agreement with both tunneling [17, 24, 28] and ARPES [11] experiments. SC coherence results from the novel pair-pair interaction, which adds a *quadron* term to the hamiltonian giving rise to a new type of fundamental excitation, the *super-quasiparticle*. The important effect is the renormalized gap function, which is energy-dependent, but non retarded.

The theory gives for the first time the correct temperature and doping dependence of the quasiparticles in the SC to PG transition. It reproduces quantitatively the experimental spectra of both pnictides and cuprates, including the peak-dip-hump structure, and attributes a common meaning to the fundamental parameters. In conclusion, these features are not due to the coupling to a bosonic mode, but rather emerge from instantaneous all-electron interactions.

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